Radicalville - A Mathematical Chalktalk

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The chalktalk, in its traditional form is an oral story accompanied by the drawing of a picture by stages at eh chalk board to illustrate the story as it unfolds. Often, regardless of the field, the chalktalk has some kind of moral. As the following chalktalk illustrates, this type of presentation can be used to teach some mathematics in a somewhat different and hopefully painless form, appropriate for either a classroom interlude or a mathematics club talk.

On the north bank of the Equal River was a small city called Radicalville. Everyone in Radicalville lived in the same kind of little house, ($\sqrt{\ }$). The people had become accustomed to their way of living and did not want to change. They felt superior to anyone who did not live as they lived. (see figure 1)

$\sqrt{20}$ $\sqrt{11}$

$\sqrt{2}$ $\sqrt{7}$ $\sqrt{13}$

Equal River

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figure 1

Some of the little houses in Radicalville had a number on the front door, and this number told something about the people who lived inside. Most of the common people of Radicalville did not have any number on their door. ($\sqrt{\ }$) These people were called square roots. However, some of the square roots wanted to pretend they were more important. the city permitted them to put a number 2 on the door. ($\sqrt[2]{\ }$) Everyone knew they were still square roots, though.

The more aristocratic people in Radicalville had larger numbers in front of their houses. These people were called cube roots ($\sqrt[3]{\ }$), fourth roots ($\sqrt[4]{\ }$) and fifth roots ($\sqrt[5]{\ }$). There were even a few sixth roots living in Radicalville. ($\sqrt[6]{\ }$) (see figure 2)

$\sqrt[2]{103}$ $\sqrt[4]{3}$

$\sqrt{20}$ $\sqrt[6]{13}$ $\sqrt{11}$

$\sqrt{2}$ $\sqrt[3]{17}$ $\sqrt{7}$ $\sqrt[2]{13}$ $\sqrt{13}$

$\sqrt[2]{3}$ $\sqrt[3]{2}$

Equal River

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figure 2

The country in which Radicalville was located was continually at war with the country located on the south side of Equal River, called the country of Exponents. Many years ago, a colony of radicals had settled south of the river and now, because of the war, they retreated back across the Equal river to live in Radicalville. Dwelling in the land of Exponents had changed these radicals. Not only did they have signs in front of their houses, but they had exponents within as well. ($\sqrt[2]{3^3}$) and ($\sqrt[3]{2^2}$). These people were resented in Radicalville, because they were considered peculiar. You could not classify these newcomers as ordinary square roots, because they had exponents in their houses, but nevertheless they were still radicals, because of their house and the sign on their door. Because they were different the lived apart.

Suddenly, Radicalville was faced with a new and more serious threat. The Land of Exponents had settled a fierce tribe of fractional exponents just across the Equal river from Radicalville, in the hope that Radicalville itself would soon be conquered. The people of Radicalville hated these fractional exponents because they did not live in houses at all. ($3^{\frac{1}{2}}$, $2^{\frac{1}{3}}$, $3^{\frac{2}{5}}$). Even in the new town of Fractional Exponents, however, there were common people and aristocrats. The greater the exponent, the more important the person. Thus, persons with exponents of 1/2 were considered more important then persons with 1/3 as and exponent.

Just as fighting was about to break out between the city of Radicalville and the town of Fractioanl Exponents, on either side of the Equal River, the Great Equater arrived on the scene. He came sailing along the Equal River in his long narrow boat made of two straight boards (=).

The Great Equator had come to teach the people of Radicalville and Fractioanl Exponents a new religion of equality. “Do not fight,” he said, “For in reality, you are all brothers. There is no difference between radicals and fractional exponents. Each of you has an equal in the other city. See, all of the square roots are rally numbers with fractional exponents, with an exponent of 1/2.

For example, $\sqrt{3} = 3^{\frac{1}{2}}$. The cube roots are really numbers with a fractional exponent of 1/3.

For example, $\sqrt[3]{7} = 7^{\frac{1}{3}}$.”

Even the strange combinations of radicals and exponents were included in the new relationships. Those who had exponents within their houses and a sign outside were equal to some person in the city of fractional exponents. For example, $\sqrt[3]{3^2} = 3^{\frac{2}{3}}$, and $\sqrt[4]{7^3} = 7^{\frac{3}{4}}$. The exponent within was the numerator of the fractional exponent, and the sign on the house was the denominator of the fractional exponent.

At first, the people in both cities were amazed, If this religion was true, then it meant that the common people of Radicalville, with or without a sign of two in front of their house were equal to the exponent of 1/2. On the other hand, the aristocrats of Radicalville, the 4^{th} , 5^{th}, and 6^{th} roots were in reality related to the poor people of the city of fractional exponents that had small exponents of 1/3, 1/4, 1/5, and 1/6. Some of the strange combinations of radicals with exponents in their houses were the most aristocratic of all, since they had larger fractional exponents such as 3/4. In fact, some of them found themselves related to numbers with integral exponents, such as $\sqrt[3]{3^6} = 3^2$.

It was not long before the people of the two cities began to think and act as one. They built huge bridges, called equations, across the Equal River, and began to intermingle. Families discovered long-lost relatives. The war was forgotten. Aristocrats and lowly people gave up their differences, since the interpretation of the difference seemed to depend entirely on the notation used.

The Great Equator sailed back along the Equal River, leaving the radicals and fractional exponents connected by their equations, and ready to work for the common mathematical good.

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