

Introduction:

The addition or subtraction of two or more trigonometric functions is called a *compound* trigonometric function or a *combination* of trigonometric functions. When graphing the combination of two functions, such as $y = \sin 2x + 3\cos x$, the calculator is adding at each x value the corresponding y values (or ordinates) of each function

Goal:

Using your graphing calculator, you are going to determine under what conditions the given compound trigonometric functions below are sinusoidal and then simplify each one into a single sine function. Recall: Sinusoidal - a function that can be written in the form of $y = a \sin(b(x + c)) + d$. (Note: In all problems on this sheet 'd', the vertical shift, will be zero.)

Preliminary Setup:

- 1) [MODE]: Be in **Radians** not Degrees!
- 2) [Window/Range]: $X_{min} = -2\pi$; $X_{max} = 2\pi$; $X_{scl} = \pi/2$; $Y_{min} = -8$; $Y_{max} = 8$; $Y_{scl} = 1$
- 3) [Y=]: See the equations below in each section

Directions:

Each person in the group is to graph in each section one or two different equations given below. Compare the graphs in the group each time one is graphed. You are observing whether or not the graph is a sinusoid i.e. a sine graph. (Note: The graphs do not have to be the same they just have to be in the shape of a sine graph with varying amplitudes, periods, and phase shifts.)

*TI81 calculators note: When graphing problems of the form $a \sin b(x + c)$ an extra set of parenthesis are needed: $a \sin (b(x + c))$

I. Changing the amplitude 'a' while keeping 'b', and 'c' the same:

- Graph: A) $y = 2\sin(x) + 3\cos(x)$ B) $y = 2\sin(2(x + 1)) + 3\cos(2(x + 1))$
 $y = 2\sin(x) - 3\cos(x)$ $y = 2\sin(2(x + 1)) - 3\cos(2(x + 1))$
 $y = 4\sin(x) + 2\cos(x)$ $y = 4\sin(2(x + 1)) + 2\cos(2(x + 1))$
 $y = 3\sin(x) - 2\cos(x)$ $y = 3\sin(2(x + 1)) - 2\cos(2(x + 1))$
 $y = 4\sin(x) - 2\cos(x)$ $y = 4\sin(2(x + 1)) - 2\cos(2(x + 1))$

- 1) Will the graphs be sinusoidal if only the amplitude number is changed? (yes/no) 1) _____
- 2) Find the following given: $y = 3\sin(x) + 2\cos(x)$ Hint: GRAPH and TRACE
 - a) amplitude (to tenths place) 2a) _____
 - b) period (in terms of π , hint: each x tic unit is $\pi/2$ long) b) _____
 - c) 'b' (hint: use the answer from b) above) c) _____
 - d) if this were a single sine function find its phase shift to the right (to 10ths place) d) _____
 - e) if this were a single sine function find its phase shift to the left (to 10ths place) e) _____
 - f) the single sine equation with a phase shift to the right f) $y =$ _____
 - g) the single sine equation with a phase shift to the left g) $y =$ _____

GRAPH the original compound equation and the answers to f) and g) at the same time to see if they are the same. (Note: Because of the rounding the graphs may be just slightly off.)

II. Changing the phase shift 'c' while keeping 'a', and 'b' the same:

Graph: $y = 3\sin(2(x + 1)) + 2\cos(2(x + 2))$ $y = 3\sin(2(x + 2)) + 2\cos(2(x + 1))$
 $y = 3\sin(2(x - 1)) + 2\cos(2(x + 1))$ $y = 3\sin(2(x + 3)) + 2\cos(2(x - 2))$
 $y = 3\sin(2(x - 2)) + 2\cos(2(x + 2))$ $y = 3\sin(2(x + 3)) + 2\cos(2(x - 3))$

- 1) Will the graphs be sinusoidal if only the phase shift number is changed? (yes/no) 1) _____
- 2) Find the following given: $y = 2\sin(2(x - 1)) + 4\cos(2(x + 2))$ Hint: GRAPH and TRACE
 - a) amplitude (to tenths place) 2)a) _____
 - b) period (in terms of Π , hint: each x tic unit is $\Pi/2$ long) b) _____
 - c) 'b' (hint: use the answer from b) above) c) _____
 - d) if this were a single sine function find its phase shift to the right (to 10ths place) d) _____
 - e) if this were a single sine function find its phase shift to the left (to 10ths place) e) _____
 - f) the single sine equation with a phase shift to the right f) $y =$ _____
 - g) the single sine equation with a phase shift to the left g) $y =$ _____

GRAPH the original compound equation and the answers to f) and g) at the same time to see if they are the same. (Note: Because of the rounding to 10ths place the graphs may be slightly off.)

III. Changing the period by changing 'b' while keeping 'a', and 'c' the same:

Graph: $y = 3\sin(4(x + 1)) - 2\cos(2(x + 2))$ $y = 3\sin(1(x + 1)) - 2\cos(4(x + 2))$
 $y = 3\sin(2(x + 1)) - 2\cos(4(x + 2))$ $y = 3\sin(1(x + 1)) - 2\cos(2(x + 2))$
 $y = 3\sin(3(x + 1)) - 2\cos(2(x + 2))$ $y = 3\sin(2(x + 1)) - 2\cos(3(x + 2))$
 $y = 3\sin(2(x + 1)) - 2\cos(3(x + 2))$ $y = 3\sin(2(x + 1)) - 2\cos(1(x + 2))$

- 1) Will the graphs always be sinusoidal if only the period is changed? (yes/no) 1) _____
- 2) Explain, using complete sentences and the words amplitude, period, & phase shift, when a compound trigonometric function will be sinusoidal.

2) _____

- 3) Find the following given: $y = 3\sin(2(x - 2)) + 3\cos(1(x - 2))$ Hint: GRAPH and TRACE
- a) amplitude (to tenths place) 3a) _____
 - b) period (in terms of Π , hint: each x tic unit is $\Pi/2$ long) b) _____
- 4) Find the following given: $y = 3\sin(0.5x) + 3\cos(2x)$ Hint: GRAPH and TRACE
- a) amplitude (to tenths place) 4a) _____
 - b) period (in terms of Π , hint: each x tic unit is $\Pi/2$ long; hint: change Xmax) b) _____

IV. Extensions:

- A) In sections I, II, and especially III on pages 1 and 2 do the following:
- i) replace the cosine by the sine so that you have a compound function of the form $a_1\sin(b_1x) + a_2\sin(b_2x)$ and
 - ii) replace the sine by the cosine so that you have a compound function of the form $a_1\cos(b_1x) + a_2\cos(b_2x)$
- 1) Explain, using complete sentences and the words amplitude, period, & phase shift, when (if at all) these compound trigonometric functions will be sinusoidal.

1) _____
